GROUP GAIN DESIGN FOR OVERLOADED CDMA

Gianmarco Romano, Domenico Ciuonzo, Francesco Palmieri, Pierluigi Salvo Rossi

Dipartimento di Ingegneria dell'Informazione,
Seconda Università di Napoli,
via Roma, 29, 81031 Aversa (CE), Italy.

ABSTRACT

In overloaded CDMA decision-feedback receivers may have a bit error probability of at least one user that "floors", i.e. cannot be reduced to zero when the signal-to-noise ratio increases. This problem is a consequence of the overloaded nature of the CDMA system, characterized by a very high correlation among users' signatures and corresponding high multiuser interference that the receiver is not able to cancel. In this paper we show that "floors" can be avoided by properly controlling each user's gain and propose an algorithm that, grouping users, sets gains in such a way that all users have a BER that vanishes. Simulations show that group decision-feedback receivers with simple conventional receivers as constituent blocks, have performances very close to those of the optimal ML receiver.

I. OVERLOADED CDMA

Consider the signal received from M users over an AWGN channel [1]

$$y(t) = \sum_{m=1}^{M} a_m b_m f_m(t) + \sigma n(t), \qquad (1)$$

where $b_m \in \{-1, +1\}$ is the information bit transmitted by the m-th user, $f_m(t)$ is the corresponding signature waveform, $a_m > 0$ is the user's signal amplitude and $n(t)$ is additive white Gaussian noise with zero mean and unit variance. A discrete-time standard model can be obtained if the received signal is projected onto a set of N orthonormal basis functions that span the signatures $\{f_m(t)\}_{m=1}^{M}$. Within this model N is also called the *spreading factor* (or processing gain) of the CDMA system. The resulting vector model is

$$\mathbf{y} \quad = \quad \mathbf{FAb} + \sigma\mathbf{n} \qquad (2)$$

where $\mathbf{y}$ is the N-dimensional observed column vector, $\mathbf{b} = (b_1, ..., b_M)^T$ is the M-dimensional binary information vector, $\mathbf{n}$ is the N-dimensional noise and the matrix $\mathbf{A} = \mathrm{diag}(a_1, ..., a_M)$ controls the energy for each user. Matrix $\mathbf{F}$ contains in each column the "signatures" for the various users. The model allows for the real-valued users' signatures to have different energies. When $M < N$ the problem is "underloaded", for $M = N$ is "fully-loaded" and for $M > N$ "overloaded".

We focus on overloaded CDMA systems because we are concerned with the strong limitations that exist with linear detection in such systems. Limitations of linear detectors where first observed by Kapur and Varanasi in [2] with reference with MMSE detection and (generalized) Welch Bound Equality signatures and then extended to any set of signatures by Varanasi *et al.* in [3]. A more general result, for binary constellations, has been found by Romano *et al.* in [4]: for any overloaded synchronous CDMA system, at least one user has a probability of error that does not vanish as the channel noise goes to zero.

It is well known that the optimal detector is a standard minimum distance classifier. It does not suffer from the "floor" problem, but has an exponential complexity in the number of users [1]. Linear detectors have limited computational complexity and are often employed in practical underloaded systems, but in overloaded CDMA are not asymptotically efficient.

Limitations of linear receivers in overloaded CDMA have an insightful geometric interpretation in terms of linear separability of sets of points. Look at Figure 1, where projections of constellation points into the signal space are shown for a system with a spreading factor $N = 2$ and a number of users $M = 5$. The $2^5 = 32$ constellation points are generated via $\mathbf{FAb}$, with $\mathbf{A} = \mathbf{I}$ and a matrix $\mathbf{F}$ chosen (arbitrarily) to be

$$\mathbf{F} = \begin{pmatrix} -0.76 & 0.89 & -0.81 & -1.0 & 0.79 \\ 0.65 & -0.46 & -0.59 & 0.03 & -0.62 \end{pmatrix}. \qquad (3)$$

The constellation points are drawn with a circle for a $+1$ and a cross for a -1 for each user. We have five pictures for the five partitions that represent the associations to the 5 bits $(b_1, ..., b_5)$. The additive Gaussian noise disperses the observations around each signal point (in a spherical fashion) and it is not shown in the figure. Linear detectors can be geometrically interpreted as hyperplanes that split the projected subspace of users' signals into two semispaces for each user. In Fig. 1 only user 3 can be linearly decoded, since a hyperplane can separate the two subsets and decode b_3; all

1-4244-0955-1/07/$25.00 ©2007 IEEE.

other users, instead, are *not* separable via a linear classifier. The asymptotic convergence of the error probability to zero as the signal-to-noise ratio grows is guaranteed when all users are separable by some hyperplane, a condition that for CDMA underloaded is frequently satisfied, but for the overloaded case cannot exist for all users simultaneously [4]. The consequence of having users that are not asymptotically linearly separable in overloaded CDMA is that the joint BER "floors" as the signal-to-noise ratio increases. This does not depend on the choice of linear receivers, users' signatures and power distribution: the limitation in the performances of overloaded CDMA with linear receivers is inherent to the overloaded nature of the problem [4].

The outline of the paper is as follows. We briefly recall decision-feedback receivers in section II and state formally geometric interpretation and properties of linear receivers in section III. The main contribution of the paper in in section IV where the group gain design algorithm is presented. In section V results from simulations are shown and commented and in section VI conclusions are drawn.

II. DECISION-FEEDBACK DETECTORS

Among non-linear detectors with limited complexity, decision-feedback detectors [5] are a good solution, since they show a good trade-off between computational complexity and performances in underloaded CDMA. However, their asymptotic efficiency need to be well studied and understood in overloaded CDMA because in some cases they might show up the "floor" problem anyhow [6], [4]. The decision-feedback receiver makes decision sequentially and uses the already acquired users to cancel progressively the multiuser interference (Fig. 2). Decision can be made one user at a time or groupwise. In the latter case the receiver is said *group decision-feedback detector*. The computational complexity is further limited if we assume that the decision about users belonging to the same group is made by a linear receiver (linear decision-feedback detector).

The simplest detector is based on linear discriminators $\mathbf{c}_i$ matched to the users' signature (i.e. $\mathbf{c}_i = \mathbf{f}_i$) with no feedback. We call this detector *conditional conventional detector* (CC detector). At the output of each linear filter a hard decision, with a threshold that depends on the previous decisions, is made and passed to the next blocks. The main advantage of CC detector is low complexity, but, for generic constellations, detectors with much better performances exist.

The choice of MMSE filters [1] as basic building blocks leads to a *conditional MMSE detector* (CMMSE detector). Hard decisions are made by each classifier and filters $\mathbf{c}_i$ are fixed. The computation complexity of such a detector is also very limited.

If the hyperplanes are computed dynamically at each step for each user as the MMSE solution on previous hard or soft decisions, we have a sort of turbo detector. The PDA receiver, for example, is inspired by such an idea [7] and performs at each stage a sequential iteration through the users computing dynamically the filters for a subset of users on the knowledge of the a posteriori probability of the others. More iterations may be necessary and the users' order may be modified dynamically. While nearly-optimal for underloaded CDMA, in the overloaded case may present poor performance [4].

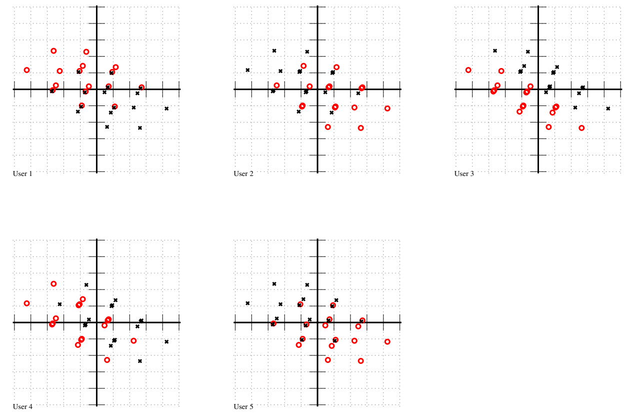

Fig. 1. Projections of the constellation points corresponding to the CDMA system with spreading factor $N = 2$, $M = 5$ users, signatures the columns of $\mathbf{F}$ as defined by eq. (3) and $\mathbf{A} = \mathbf{I}$.

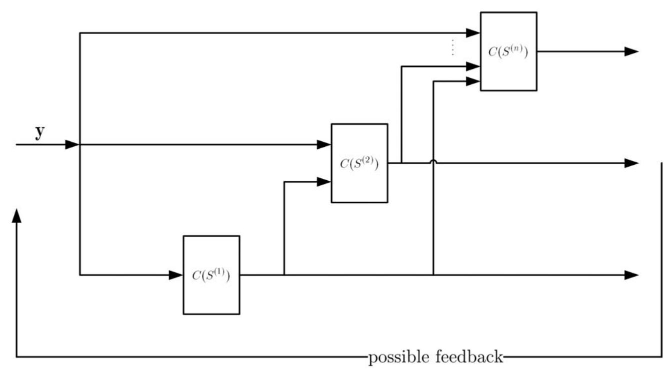

Fig. 2. The group decision-feedback structure for a CDMA system.

III. LINEAR SEPARABILITY

We state now more formally the idea of user linear separability [4]. We indicate the set subtraction operator by the minus sign, e.g. $\mathcal{U} - \{i\}$ is the set $\mathcal{U}$ without its element i.

Definition 1: Let $\mathcal{U} = \{1, 2, ..., M\}$ be a set of users. User $i \in \mathcal{U}$ is *asymptotically linearly separable* (or simply *linearly separable*) if for $\sigma^2 \to 0$ there exists an N-

dimensional vector $\mathbf{c}_i = (c_{1i}...c_{Ni})^T$:

$$\left\{ \begin{array}{ll} \mathbf{c}_i^T \mathbf{y} > 0 & \text{if } b_i = 1 \\ \mathbf{c}_i^T \mathbf{y} < 0 & \text{if } b_i = -1 \end{array} \right. \forall\, b_j \in \{-1, +1\}\,,\ j \neq i. \quad (4)$$

Under linear detection and with the decision rule $\hat{b}_i = \mathrm{sgn}\left(\mathbf{c}_i^T \mathbf{y}\right)$ for each user i, it can be proved that the users' geometrical separability is a sufficient and necessary condition to have a vanishing single-user BER. An alternative equivalent condition is provided by the following theorem (for the proof see [4]).

Theorem 1: If user i is *asymptotically linearly separable*, there exists an N-dimensional vector $\mathbf{c}_i = (c_{1i}...c_{Ni})^T$ that satisfies the following condition

$$a_i |\mathbf{c}_i^T \mathbf{f}_i| > \sum_{j \in \mathcal{U} - \{i\}} a_j |\mathbf{c}_i^T \mathbf{f}_j| \quad (5)$$

In reference to the normalized linear filters $\mathbf{v}_i = \mathbf{c}_i / \sqrt{\mathbf{c}_i^T \mathbf{c}_i}$, we define the *linear margin* as

$$\delta_i = a_i |\mathbf{v}_i^T \mathbf{f}_i| - \sum_{j \in \mathcal{U} - \{i\}} a_j |\mathbf{v}_i^T \mathbf{f}_j|. \quad (6)$$

Therefore user i, to be asymptotically separable, must have a positive margin. The linear margin is strictly related to another parameter of interest in CDMA systems, which is the asymptotic effective energy (AEE), as defined by Varanasi in [5], for linear receivers. Denote with E_i the AEE for user i, then

$$E_i = \max{}^2 \{0, \delta_i\}\,. \quad (7)$$

Therefore δ_i^2 represents the AEE when user i is separable. E_i is zero when condition (5) does not hold, and consequently $E_i > 0$ becomes a sufficient and necessary condition for separability. At least in the case of linear receivers, the linear margin represents a sort of distance from the separability condition to hold, i.e. it can tell how far from separability one user is.

By defining $\mathbf{H} = \mathbf{V}^T \mathbf{F} \mathbf{A}$, where $\mathbf{V}$ is the $N \times M$ matrix that has as columns the normalized linear filters $\mathbf{v}_i$, $i = 1, 2, \ldots, M$, separability condition for user i can be rewritten as

$$|h_{ii}| > \sum_{j \neq i} |h_{ij}|\,, \quad (8)$$

where h_{ij} is the generic element of the $M \times M$ matrix $\mathbf{H}$: *user i is linearly separable if the i-th row of $\mathbf{H}$ is diagonally dominant.*

In overloaded CDMA matrix $\mathbf{H}$ is cannot be fully diagonally dominant [4]. In general a user can become asymptotically efficient by increasing its power, i.e. increasing a_i, but this can affect negatively the others. In overloaded CDMA some users are affected so much by multiuser interference that it is impossible to have a probability of error that asymptotically goes to zero for all users at the same time.

The group detection in linear decision-feedback receivers assumes that decisions on previous groups are already made and the group probability of error asymptotically goes to zero if users in the group are linearly separable. As an example, look at the constellation in Fig. 1. After a decision on user 3 has been taken, a linear detector may be used to make a decision on user 4, but not on the others. Further conditioning is necessary.

Definition 2: Consider a subset $\mathcal{G} \subset \mathcal{U}$ of users and suppose that the bits for the users in $\mathcal{G}$ are all known, i.e. $\left\{b_j = b_j^*, j \in \mathcal{G}\right\}$. Then define

$$\mu_{\mathcal{G}}^* = \sum_{l \in \mathcal{G}} a_l \mathbf{f}_l b_l^*. \quad (9)$$

We say that user $i \in \mathcal{U} - \mathcal{G}$ is *linearly separable, conditionally on $\mathcal{G}$,* if for $\sigma^2 \to 0$, there exists an N-dimensional vector $\mathbf{c}_i = (c_{1i}...c_{Ni})^T$:

$$\begin{cases} \mathbf{c}^T(\mathbf{y} - \mu_{\mathcal{G}}^*) > 0 & \text{if } b_i = 1 \\ \mathbf{c}^T(\mathbf{y} - \mu_{\mathcal{G}}^*) < 0 & \text{if } b_i = -1 \end{cases}$$
$$\forall\, b_j \in \{-1, +1\}\,,\ j \in \mathcal{U} - \mathcal{G} - \{i\} \quad (10)$$

Therefore a conditionally linearly separable user can be detected with a linear classifier after the users in $\mathcal{G}$ have been "subtracted out". The decision rule for user i becomes $\mathrm{sgn}\left(\mathbf{c}_i^T \left(\mathbf{y} - \mu_{\mathcal{G}}^*\right)\right) \gtrless 0$.
The condition on matrix $\mathbf{F}$ and the gains in $\mathbf{A}$ is similar to the unconditioned case and it is described by the following extension of Theorem 1.

Theorem 2: User i is *linearly separable conditionally on $\mathcal{G}$* if there exists an N-dimensional vector $\mathbf{c}_i = (c_{1i}...c_{Ni})^T$:

$$a_i |\mathbf{c}_i^T \mathbf{f}_i| > \sum_{j \in \mathcal{U} - \mathcal{G} - \{i\}} a_j |\mathbf{c}_i^T \mathbf{f}_j|. \quad (11)$$

Similarly to the unconditional case, we define the *conditional linear margin*

$$\delta_{\mathcal{G}i} = a_i |\mathbf{v}_i^T \mathbf{f}_i| - \sum_{j \in \mathcal{U} - \mathcal{G} - \{i\}} a_j |\mathbf{v}_i^T \mathbf{f}_j|, \quad (12)$$

IV. GROUP GAIN DESIGN ALGORITHM

We have already shown that by simply changing the gain parameters in $\mathbf{A}$, as for example in systems where some power control mechanism is implemented, we can achieve conditional separability [4]. We assume that we know the signatures $\mathbf{F}$, and that they cannot be changed. They may be imposed by channel parameters or other constraints. The possibility and capability of changing the gains is crucial especially for overloaded CDMA systems because, as we show in the following, by choosing appropriately the set of gains any set of M users with any set of signatures $\mathbf{F}$ can always be made conditionally linearly separable. In other words it is possible by adjusting the gains to make a linear group decision-feedback receiver asymptotically efficient even for overloaded CDMA. The algorithm will prove useful also for underloaded CDMA systems (they do not generally suffer from the "floor" problem), because adjusts the gains such that we can improve asymptotic efficiency by imposing appropriate linear (conditional) margins.

The group gain design algorithm proposed here, generalizes the algorithm in [4] by setting users' gains groupwise instead of each user at time. The algorithm follows these steps:

1) choose all δ_i's, $i \in \mathcal{U}$;
2) choose any subset of $N^{(1)} \leq N$ users, $\mathcal{I}^{(1)} = \left\{ i_1^{(1)}, \dots, i_{N^{(1)}}^{(1)} \right\}$, whose signatures are linearly independent;
3) set $\mathcal{G}^{(1)} = \mathcal{I}^{(1)}$;
4) for each user $i \in \mathcal{I}^{(1)}$ set the corresponding gains by solving the linear system:

$$a_i \left| \mathbf{v}_i^T \mathbf{f}_i \right| - \sum_{j \in \mathcal{G}^{(1)} - \{i\}} a_j \left| \mathbf{v}_i^T \mathbf{f}_j \right| = \delta_i \qquad (13)$$

5) set $n = 2$;
6) find any subset of $N^{(n)} \leq N$ users, whose signatures are linearly independent, $\mathcal{I}^{(n)} = \left\{ i_1^{(n)}, \dots, i_{N^{(n)}}^{(n)} \right\}$;
7) set $\mathcal{G}^{(n)} = \mathcal{G}^{(n-1)} \cup \mathcal{I}^{(n)}$;
8) for each user $i \in \mathcal{I}^{(n)}$ set the gains as

$$a_i \left| \mathbf{v}_i^T \mathbf{f}_i \right| - \sum_{j \in \mathcal{G}^{(n)} - \{i\}} a_j \left| \mathbf{v}_i^T \mathbf{f}_j \right| = \delta_i \qquad (14)$$

9) set $n = n + 1$ and repeat steps 6), 7) and 8) until all users have been included in $\mathcal{G}^{(n)}$;

The algorithm sets jointly the gains of all users belonging to the same group. The size of the group has a maximum that can be no greater than N. This is because for overloaded CDMA we cannot have more than N users (conditionally) linearly separable. In general it might be smaller than N and is a design parameter. Having groups of size greater than 1 reduces the number of conditioning in the decision-feedback detector and thus improves its performances, when hard decisions are made, since error propagation is limited.

The idea is that by conditioning we decompose the detection problem of an overloaded CDMA into the detection of more underloaded CDMA systems. Consider the structure shown in Fig. 2: each linear block makes a (soft or hard) decision on a set of users based on the decision on another subset of users (and the received signal); each block represents the detector for an underloaded CDMA. For the receiver to succeed there must exist a users order for which each underloaded subsystem is asymptotically efficient. The group gains design algorithm reverses the conditioning of the groups in setting the gains because the setting of the margin for user i requires the gains for the users already examined and the known output values for all the others. The imposed conditional margins $\{\delta_i\}$, $i \in \mathcal{U}$, are design parameters of the algorithm and they can be regulated to increase as much as possible the asymptotic efficiency.

The algorithm clearly depends on the specific choice of linear filters. For example, in the case of MMSE filters (CMMSE design), we have that the matrices of coefficients of the linear systems (13) and (14) that must be solved to get the gains are diagonals. To see this, consider the linear system at step 4) of the algorithm, that can be express in matrix form as$\mathbf{T}\mathbf{a} = \delta$. Note that the matrix $\mathbf{T}$ can be obtained from $\mathbf{V}^T\mathbf{F} = \mathbf{D}^{-1/2}\mathbf{C}^T\mathbf{F}$, and that, for MMSE detectors in underloaded CDMA, $\mathbf{C}^T\mathbf{F}\mathbf{A} = \mathbf{I}$. Therefore we obtain that $\mathbf{D}^{-1/2}\mathbf{C}^T\mathbf{F} = \mathbf{D}^{-1/2}\mathbf{A}^{-1}$, which is diagonal and thus also $\mathbf{T}$ is diagonal.

When the linear filters are chosen to be the signatures (CC design), the matrix $\mathbf{T}$ is diagonal only when the signatures associated to the users in the same groups are orthogonal. In general the crosscorrelation among signatures determines the presence of non diagonal elements. Note that matrix $\mathbf{T}$ is symmetric and the solutions of the linear systems need to be positive. The determination of all necessary and sufficient conditions on $\mathbf{T}$ for the solution to be positive are based on the structure of the matrix and will be reported elsewhere.

V. SIMULATIONS

We implemented and used the above algorithm in a simulated scenario of $M = 5$ users and processing gain $N = 2$, with a group decision-feedback detector and decisions made within each group by conventional receivers (CC detectors). The set of signatures are randomly generated and kept fixed for all the simulations. We have compared two sets of gains, obtained with the proposed algorithm, with the system with all equal gains: one with groups of size one; the other with groups of size N. All sets of gains are subject to the same sum-power constraint and the (conditional) linear margin are set to 1.

Results are reported in Figs. 3, 4 and 5, where performances in terms of joint BER versus average energy per bit-to-noise ratio are shown. More specifically in Fig. 3 we show that the system under consideration with no gain design suffers from the BER "floor" for the CC detector, but does not for the CMMSE detector. When gains are designed according our algorithm, as shown in Figs. 4 and 5, both detectors are asymptotically efficient. Since we change the gains distribution we have also an improvement with the optimum receiver. The group size is a design parameter of the algorithm, we have run simulations for two values of group size. We note that when the group size is equal to its maximum value, i.e. the processing gain N, the CC detector performs very well and quite close to the ML receiver, having the great advantage of having a low computational complexity. In all the results the CMMSE detector performances (with the same set of gains) show that our algorithm can be used to improve the performances of an already asymptotically efficient receiver and that the filters used in the algorithm do not need to be the same of those

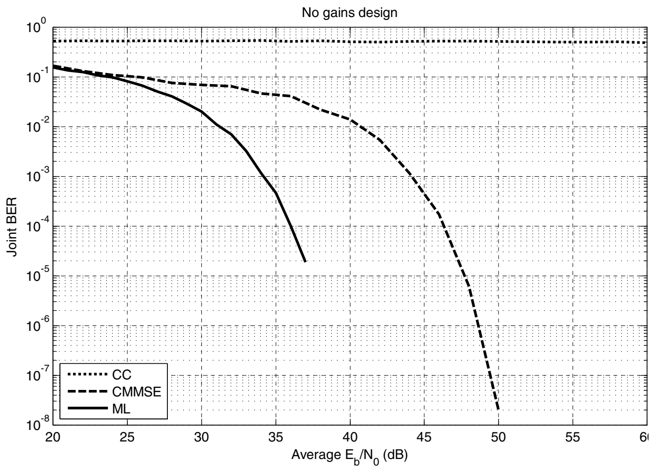

Fig. 3. Simulation results a CDMA system with spreading factor $N = 2$ and $M = 5$ users and *no gain design* ($\mathbf{A} = \mathbf{I}$).

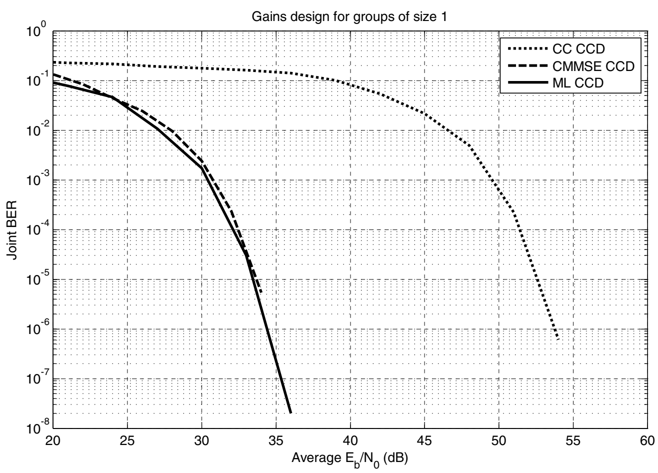

Fig. 4. Simulation results for a CDMA system with spreading factor $N = 2$ and $M = 5$ users. Gains are obtained from the gain design algorithm with $\mathbf{v}_i = \mathbf{f}_i$ (CC design) and group sizes equal to 1.

employed by the receiver. With any group size the CMMSE is able to have nearly-optimal performances.

VI. CONCLUSIONS

In overloaded CDMA, the joint BER might not go to zero as the signal-to-noise ratio increase. While this cannot be avoided with linear receivers, non-linear detectors like linear decision-feedback receivers need careful gain control to avoid "floors". We propose an algorithm that sets the gains groupwise in a synchronous CDMA system in such a way that the joint BER reduces when the channel noise goes to zero. Simulations show that a conventional decision-feedback detector not only can be made asymptotically efficient, but also that its performances are very close to the optimum, despite its low computational complexity.

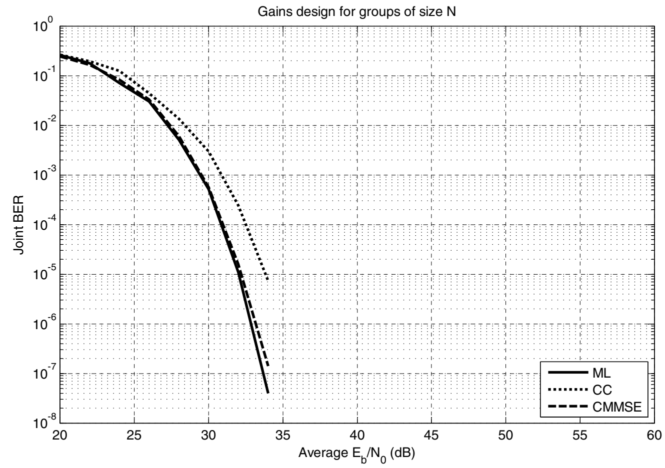

Fig. 5. Simulation results for a CDMA system with spreading factor $N = 2$ and $M = 5$ users. Gains are obtained from the gain design algorithm with users' signatures as hyperplanes, i.e. $\mathbf{v}_i = \mathbf{f}_i$ (CC design) and group sizes equal to N.

VII. REFERENCES

[1] S. Verdú, *Multiuser Detection*. Cambrige University Press, 1998.

[2] A. Kapur and M. Varanasi, "On the limitation of linear MMSE detection and (Generalized) Welch Bound Equality signals," in *Proc. of Conference on Information Sciences and Systems*, The Johns Hopkins University, March 12–14 2003.

[3] M. Varanasi, C. Mullis, and A. Kapur, "On the limitation of linear MMSE detection," *IEEE Transactions on Information Theory*, vol. 52, pp. 4282–4286, Sept. 2006.

[4] G. Romano, F. Palmieri, P. K. Willett, and D. Mattera, "Asymptotic multiuser efficiency in overloaded CDMA," in *Proc. of the 40th Conference on Information Sciences and Systems (CISS)*, (Princeton, NJ, USA), Princeton University, March 2006.

[5] M. K. Varanasi, "Decision feedback multiuser detection: a systematic approach," *IEEE Transactions on Information Theory*, vol. 45, pp. 219–240, Jan. 1999.

[6] A. Kapur and M. K. Varanasi, "Multiuser detection for overloaded CDMA systems," *IEEE Transactions on Information Theory*, vol. 49, pp. 1728–1742, Jul. 2003.

[7] J. Luo, K. R. Pattipati, P. K. Willett, and F. Hasegawa, "Near-optimal multiuser detection in synchronous CDMA using probabilistic data association," *IEEE Communications Letters*, vol. 5, pp. 361–363, Sept. 2001.

[8] G. Romano, F. Palmieri, D. Mattera, and P. K. Willett, "Separability and gain control for overloaded CDMA," *submitted to IEEE Transactions on Information Theory*, vol. -, pp. –, 2006.